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Modeling Volatility Persistence and Asymmetry with Exogenous Breaks in the Nigerian Stock Returns

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This study examines the volatility persistence and asymmetry with exogenous breaks in Nigerian stock market. The study utilizes daily closing quotations of stock prices from the Nigerian stock exchange for the period 3rd July, 1999 to 12th June, 2017. Standard symmetric GARCH (1,1), asymmetric EGARCH (1,1) and GJR-GARCH (1,1) models with and without structural breaks were employed to measure shocks persistence and leverage effects in the presence of varying distributional assumptions. The empirical findings showed high persistence of shocks in the return series for the estimated models. However, the study found significant reduction in shocks persistence when structural breaks were incorporated in the estimated models. Empirical evidence for the existence of asymmetry without leverage effect was found in Nigerian stock market. The EGARCH (1,1) model with student-t innovation density was found to fit the data better than other competing models. The study recommends the incorporation of structural breaks while estimating volatility in the Nigerian stock market. This will help to avoid over-estimation of volatility shocks and restore investor's confidence in the stock market.

Keywords: Asymmetry; GARCH Family Models; Leverage Effect; Nigeria; Shock Persistence; Volatility.
JEL Classification: C22, C32, C52.

1.0 Introduction

Volatility modeling of stock market returns has been gaining great interest by financial markets participants, academia, financial analysts and the general public. Volatility measures the uncertainty and risk which play significant role in modern financial analysis. Measuring and predicting volatility is crucial for financial decision making and has significant applications in areas such as portfolio selection, option pricing, risk management, hedging and strategic pair-trading as well as Value-at-Risk (VaR) estimation.

Providing accurate volatility estimates in Nigeria market avails regulators,

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government, traders and investors the opportunity to formulate better policies and make appropriate financial investment decisions. However, the accuracy of volatility estimates and the precision of interval forecast is compromised if structural breaks are ignored (Orabi and Alqurran, 2015; Adewale et al., 2016; Kuhe and Chiawa, 2017). Studies conducted by Perron (1989, 1990)); Diebold and Inoue (2001); Yaya, Gil-Alana and Shittu (2005), among others, revealed that when stationary processes are contaminated with structural breaks, the sum of Autoregressive Conditional Heteroscedasticity (ARCH) and Generalized ARCH (GARCH) terms are always biased to unity. It is therefore, reasonable to incorporate these sudden shifts in variance when modeling and estimating parameters of volatility models. In our opinion, if the causes of shocks in stock prices are identified and accounted for, it will yield more reliable and accurate volatility estimates, this could also help in making good financial reforms that may have positive impacts and direct bearing on financial institutions and the economy.

This paper discusses some of the modern methods of modeling and estimating volatility of stock returns with application on daily financial time series from the Nigerian stock market. One of the important aspects that must be put into consideration while modeling volatility of financial time series data such as stock returns is that such series exhibits some regular patterns called statistical regularities or stylized facts which are well recognized and documented in the literature and which are crucial for correct model specification and parameter estimation. The objectives of this study are therefore in two-folds: (i) to examine the persistence of shocks and asymmetric response in the Nigerian stock market; (ii) to investigate the impact of exogenous breaks on the conditional variance in Nigerian stock returns. This study shall employ Bai and Perron multiple structural breakpoints testing procedure that detects breakpoints in the entire data set of the Nigerian stock returns. Once the breakpoints are detected, they can be accounted for by creating an indicator variable which takes the value zero for stable and one for unstable regimes. These breakpoints will be incorporated in the variance equation of all symmetric and asymmetric GARCH models to avoid over persistence of volatility shocks in the conditional variance.

The rest of the paper is organized as follows: Section 2 reviews relevant literature on the subject matter, Section 3 presents data and methodology; Section 4 discusses results of empirical findings while Section 5 hinges on conclusion and policy implications.

2.0 Literature Review

Modeling volatility of stock market return series using time varying GARCH models proposed by Engle (1982), Bollerslev (1986) and extended by Nelson (1991), Glosten et al. (1993), Ding et al. (1993), Zakoian (1994), etc. has been gaining attention in recent times by policy makers, academics, financial analysts and researchers among others. This is partly as a result of the fact that GARCH family models have been more successful in capturing stylized facts (statistical regularities) of financial time series such as volatility clustering, volatility shock persistence, volatility mean reversion, leverage effect and risk premium among others; and partly because volatility is an important concept for many economic and financial applications such as risk management, option trading, portfolio optimization and asset pricing. The prices of stocks and other assets depend on the expected volatility of returns. As part of monitoring risk exposure, banks and other financial institutions make use of volatility assessments (Engle and Patton, 2001).

Recent studies have shown that estimates of stock returns volatility are considerably affected by sudden structural breakpoints or sudden regime shifts which occur as a result of domestic and external shocks(Kumar and Maheswaran, 2012).

Many scholars have documented evidence relating to volatility models in the presence of structural breaks across developed and emerging economies. Lamoureux and Lastrapes (1990) found out that not incorporating structural breakpoints in the conditional variance while modelling volatility increases persistence of volatility shocks whereas incorporating the sudden shifts in conditional variance reduces the persistence of shocks in volatility models. In a similar vein, Malik et al. (2005) investigated the persistence of volatility shocks on the Canadian stock data using heteroskedastic models. Results showed reduction in shocks persistence in volatility when structural break points were incorporated in the conditional variance while estimating volatility. Hammoudeh and Li (2008) also found significant decrease in volatility shock persistence when valid sudden shifts in variance were incorporated while predicting volatility in Gulf Arab countries stock markets. Muhammad and Shuguang (2015) investigated the impact of random level shifts in conditional volatility and variance persistence of GARCH family models while employing Bai and Perron multiple breakpoints test procedure to detect structural breakpoints in conditional variance of daily stock returns of 7 emerging markets from 1977 to 2014. They estimated asymmetric EGARCH (1,1) and TGARCH (1,1) with and without breaks and found that persistency in the conditional variance significantly reduced when level shifts were considered in the conditional volatility of these models. The half-lives to shocks were also found to decline significantly in the presence of these sudden break points.

In emerging economies like Nigeria, empirical evidence relating to volatility modeling with structural breaks are also documented in the literature. For example, Dikko et al. (2015) modeled abrupt shift in time series using dummy variable by employing both symmetric and asymmetric GARCH models with and without sudden shifts in variance. They used daily quotations of 10 insurance stocks of the Nigerian stock exchange from 02/01/2006 to 26/05/2014. They found significant reduction in shock persistence in volatility of most insurance stock returns when the regime shifts were incorporated into the models. Bala and Asemota (2013) employed GARCH models with and without breaks to examine the volatility of exchange rate of naira against US Dollar, British Pounds and European Euro using monthly exchange rate data. There was high persistence of shocks in all the models, although the introduction of structural breakpoints improved the volatility estimates by reducing shock persistence in most of the estimated models.

Asymmetric property was not evidenced in most estimated models. Salisu and Fasanya (2012) examined the relative performance of symmetric and asymmetric GARCH models for West Texas Intermediate (WTI) daily closing oil prices by considering the pre-crises, during crises and the post global financial crisis periods. The WTI oil stock price was found to be most volatile during the financial crises period than other sub-periods. Asymmetric models found empirical evidence for the existence of leverage effects and were found superior over the symmetric GARCH model. The study concluded that ignoring these effects in modeling oil price volatility will lead to misleading results and serious biases.

Adewale et al. (2016) investigated shock persistence and asymmetry in Nigerian stock market by incorporating structural breaks using monthly stock returns for the period from January 1985 to December 2014. They segmented the study period into pre-structural break period and after break period having identified breakpoints in the series. Result from the basic GARCH model showed higher shock persistence during pre-break sub-period than the post-break sub-period. No evidence of asymmetry or leverage effect was found in the asymmetric GARCH model with or without incorporating the breakpoints in Nigerian stock market. Recently, Fasanya and Adekoya (2017) investigated the performances of different GARCH models while estimating the volatility of headline and core CPI inflation in Nigeria for the period 1995M01 to 2016M10 using ADF breakpoint testing procedure. They applied both symmetric and asymmetric GARCH variants and observed empirical evidence of shock persistence in both CPI stock returns with the presence of leverages only in the headline CPI return series. The authors concluded that ignoring the role of structural breaks in estimating the volatility of inflation rate in Nigeria will amount to misleading policy prescriptions.

Gil-Alana et al. (2015) employed fractional integration and structural break procedures in studying the daily share prices of the Nigerian banking sector between 2001 and 2012. The results obtained through parametric and semi-parametric methods indicated little evidence of mean reversion in the return series. There was evidence of long memory in the absolute and squared return series. The presence of structural breaks was also evident with the number of breaks depending on the bank examined. The breaks which were more noticed in the month of December 2008 relating to the global financial crisis also affected the Nigerian banking sector. Kuhe and Chiawa (2017) examined the impact of structural breaks on the conditional variance of daily stock returns of 8 commercial banks in Nigerian stock market for the period 17th February, 2003 to 31st September, 2016. They employed symmetric GARCH, asymmetric EGARCH and TGARCH models with and without dummy variables to evaluate variance persistence, mean reversion, asymmetry and leverage effects. Results showed high persistence in conditional volatility for the banking stocks when the shift dummies were ignored, but when the random level shifts were incorporated into the models, there was reduction in the conditional volatility of these models. The half-lives of volatility shocks also reduced in the presence of regime shifts.

From the reviewed literature, it is glaring that authors who examined shock persistence in conditional variance in Nigerian stock market in the presence of exogenous breaks using either daily or monthly all share index (ASI) from Nigerian stock exchange segmented the data into sub-periods either as pre-crises period (pre-breaks), during crises period (break period) and post crises period (post breaks period). However, it is not only the well-known banking reform of 2004 in Nigeria and the global financial crises which started from 2007 to 2009 that can affect the Nigerian stock market. Other internal factors can also affect the stock market (Muhammad and Shuguang, 2015). This study therefore, extends the existing literature by investigating the symmetric and asymmetric responses in shocks persistence on the conditional variance of daily stock return of Nigerian stock market using both symmetric and asymmetric GARCH type models with and without structural breakpoints with more current data. This study uses methodology slightly different from the ones mentioned in the literature as it employs Bai and Perron multiple breaks testing procedure that detects breakpoints in the entire data set of the Nigerian stock returns. Once the breakpoints are detected, they are accounted for by creating an indicator (dummy) variable which takes the value zero for stable and one for unstable economy which is incorporated in the variance equation of all GARCH models to avoid over persistence of shock in the conditional variance.

3.0 Data and Methodology

3.1 Data

The data used in this research work are the daily closing all share index (ASI) of the Nigerian Stock Exchange (NSE) obtained from www.nse.ng.org for the period 03/07/1999 to 12/06/2017 making a total of 4726 observations. The daily returns r_t are calculated as:

$$r_t = 100. \ln \Delta p_t \tag{1}$$

where r_t denotes the stock return series, Δ is the first difference operator and P_t denotes the closing market index at the current day (t).

3.2 Ng and Perron (NP) modified unit root test

To check the stationarity properties of the daily stock prices and returns, Ng and Perron modified unit root test is employed because of its good power property. Ng and Perron (2001) constructed four test statistics which are based on the Generalized Least Squares detrended series ${Y_t}^d$.The four test statistics are the modified forms of Phillips and Perron Z_α and Z_t statistics, the Bhargava (1986) R_1 statistic, and the Elliot, Rothenberg and Stock Point Optimal statistic (Elliot et al., 1996). First, define the term:

$$k = \sum_{t=2}^{T} \frac{(Y_{t-1}^d)^2}{T^2} \tag{2}$$

The four modified statistics are then written as,

$$MZ_\alpha^d = (T^{-1}(Y_T^d)^2 - f_0)/(2K)$$

$$MZ_t^d = MZ_\alpha \times MSB$$

$$MSB^d = (k/f_0)^{0.5}$$

$$MP_T^d = \left\{ \begin{array}{ll} \left(-7^2k + 7T^{-1}\left(Y_T^d\right)^2\right)/f_0 & \text{if } x_t = \{1\} \\ \left(-13.5^2k + (1+13.5)\,T^{-1}\left(Y_T^d\right)^2\right)/f_0 & \text{if } x_t = \{1, t\} \end{array} \right\} \quad (3)$$

where $MZ_\alpha{}^d$ is the modified detrended Z_α transformation of the standardized estimator given by:

$$T(\hat{\alpha} - 1) = \{T^{-1}\sum_{t=1}^{T}(y_t - y_{t-1})\}/\{T^{-1}\sum_{t=1}^{T} y_t^2 - 1\} \quad (4)$$

MZ_t^d is the modified detrended Z_t transformation of the conventional regression t statistic defined by:

$$t_\alpha = (\sum_{t=1}^{T} y_{t-1}^2)^{\frac{1}{2}}(\hat{\alpha} - 1)/s \quad (5)$$

where

$$s^2 = T^{-1}\sum_{t=1}^{T}(y_t - \hat{\alpha}y_{t-1})^2 \quad (6)$$

MSB is the modified Bhargava R_1 statistic (Stock, 1990). The R_1 statistic is given by:

$$R_1 = \sum_{t=2}^{T}(y_t - y_{t-1})^2/\sum_{t=1}^{T}(y_t - \bar{y})^2; \bar{y} = \frac{1}{T}\sum_{t=1}^{T} y_i \quad (7)$$

MP_T^d is the ERS modified detrended point optimal statistic (Elliot et al., 1996). The point optimal statistic is given as:

$$P_T = \frac{[S(\hat{\alpha}) - \hat{\alpha}S(1)]}{s^2} \quad (8)$$

Y_T^d is the trended series, x_t is a series of observations at time t, f_0 is the frequency zero spectrum define as:

$$f_0 = \sum_{j=-(T-1)}^{T-1} \hat{\gamma}(j).K(\frac{j}{l}) \quad (9)$$

Where l is a bandwidth parameter, T is the sample size, K is a kernel function and $\hat{\gamma}(j)$ is the $j-th$ sample autocovariance of the residuals $\hat{u}_t$ and is given by:

$$\hat{\gamma}(j) = \sum_{t=j+1}^{T} (\hat{u}_t \hat{u}_{t-j})/T \tag{10}$$

The MZ_{α},MZ_t,MSB and MP_T statistices are collectively referred to as M tests and are used in detecting the presence of unit root in a series (Ng and Perron, 2001). In addition to the MZ_{α} and MZ_t statistics, Ng and Perron also investigated the size and power properties of the MSB statistic. Critical values for the demeaned and detrended case of this statistic were taken from Stock (1990).

3.3 Test for Heteroskedasticity

Test for heteroskedasticity (or ARCH effect) was conducted using the Lagragian Multiplier test proposed by Engle (1982). The test checks the pair of hypotheses $H_0 : \rho_1 =...= \rho_m$ versus $H_1 : \rho_1 \neq 0$ for some $i\varepsilon\{1, \ldots, m\}$. The F-statistic is estimated as:

$$F = \frac{SSR_0 - SSR_1/m}{SSR_1(n - 2m - 1)} \tag{11}$$

where,

$$SSR_1 = \sum_{t=m+1}^{T} e_t^2, SSR_0 = \sum_{t=m+1}^{T} (a_t^2 - \varpi)^2 and\ \varpi = \frac{1}{n} \sum_{t=1}^{T} a_t^2 \tag{12}$$

$\hat{e}_t$ is the residual obtained from least squares linear regression, ϖ is the sample mean of a_t^2. The ARCH LM test statistic is distributed asymptotically as chi- square distribution with m degrees of freedom under the H_0. The decision is to reject the null hypothesis of no ARCH effect in the residuals if the p-value of F-statistic is less than $\alpha = 0.05$.

3.4 Bai and Perron Multiple Breakpoints Test

Bai and Perron(1998) developed a multiple structural breakpoints testing procedure which predict persistently several shifts in variance. The power of the test was strengthened by Bai and Perron (2003) which made the test more efficient. The model considered is the multiple linear regression model with m breaks or $m + 1$ regimes.

$$y = x_i^T \beta_i + u_t \tag{13}$$

$$y_i = x_i^T \beta + z_i^T \delta + u_t \tag{14}$$

where $u_i \sim$ iid(0,σ^2),$i = 1, 2, 3, \ldots, n$ and y_i is the response variable at time i and $x_i = [1, x_{i2}, x_{i3}, \ldots, x_{ik}]^T$ is a vector of order $k \times 1$ of independent variables having one as its initial value and β_i is also $k \times 1$ vector of coefficients.

The hypothesis for random level shift is:$H_0 : \beta_i = \beta_0$ for $i = 1, 2, 3, \ldots, n$ (i.e., there is no structural break in the series) versus alternative that with the random level shift in time the vector of coefficients also changes, also assuming that they have no stochastic behaviour as a departure from the null hypothesis. i.e.,

$$\|x_i\| = O(1) \text{ and that } \frac{1}{n} \sum_{i=1}^{n} x_i x_i^T \to Z$$

where Z represents a finite matrix. This expression permits the detection of multiple breakpoints in data and once the breakpoints are recognized, they will be incorporated into each GARCH model in order to avoid spurious results. This same procedure is implemented in this study to detect multiple break points in the given stock return series before moving forwards.

3.5 Model Specification

The following conditional heteroskedasticity models are specified for this study.

3.5.1 The Autoregressive Conditional Heteroskedasticity (ARCH) Model

The ARCH model was first developed by Engle (1982). For the log return series (r_t), the ARCH (p) model is specified as:

$$r_t = \mu + \varepsilon_t \tag{15}$$

$$\varepsilon_t = \sqrt{h_t u_t}, \quad u_t \sim N(0,1) \tag{16}$$

$$h_t = \omega + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 \tag{17}$$

where r_t is the return series, ε_t is the innovation or shock at day t which follows heteroskedastic error process, μ is the conditional mean of (r_t), h_t is the volatility (conditional variance) at day t and ϵ_{t-1}^2 is the square innovation at day $t-i$. For an ARCH (p) process to be stationary, the sum of ARCH terms must be less than one (i.e., $\sum \alpha_i < 1$).

3.5.2 The Generalized Autoregressive Conditional Heteroskedasticity (GARCH) Model

The ARCH model was extended by Bollerslev (1986) called Generalized Autoregressive Conditional Heteroskedasticity (GARCH) Model. Assuming a log return series $r_t = \mu + \varepsilon_t$ where ε_t is the error term at time t.The ε_t follows a GARCH (1,1) model if:

$$h_t = \omega + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1} \tag{18}$$

with constraints $\omega > 0$,$\alpha_1 \geq 0$ and $\beta_1 \geq 0, j = 1$;$\alpha_1 + \beta_1 < 1$ to ensure conditional variance to be positive as well as stationary. The basic GARCH (1,1) model is adequate in capturing all volatility in any financial time series. The GARCH (1,1) model with dummy variable in the conditional variance is specified as:

$$h_t = \omega + \phi_1 D_1 + ... + \phi_n D_n + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1} \tag{19}$$

where D_1,$\ldots$,D_n are dummy variables added to the conditional variance equation which takes value 1 as the sudden break appears in conditional volatility onwards and otherwise it takes value 0.

3.5.3 The Exponential GARCH (EGARCH) Model

The EGARCH model was extended by Nelson (1991) to capture asymmetric effects between positive and negative stock returns. The conditional variance equation for EGARCH (1,1) model specification is given as:

$$\ln(h_t) = \omega + \alpha_1 \mid \frac{\varepsilon_{t-1}}{h_{t-1}} \mid + \gamma [\frac{\varepsilon_{t-1}}{h_{t-1}}] + \beta_1 \ln(h_{t-1}) \tag{20}$$

where γ represents the asymmetric coefficient in the model, β_1 coefficient represents the measure of shock persistence.Asymmetry exists if $\gamma \neq 0$, there is leverage effect if $\gamma < 1$. The EGARCH (1,1) model with dummy variable in the conditional variance is specified as:

$$\ln(h_t) = \omega + \phi_1 D_1 + ... + \phi_n D_n + \alpha_1 \mid \frac{\varepsilon_{t-1}}{h_{t-1}} \mid + \gamma [\frac{\varepsilon_{t-1}}{h_{t-1}}] + \beta_1 \ln(h_{t-1}) \tag{21}$$

where D_1,. . . ,D_n are the dummy variables which take the value 1 for each point of sudden change in variance onwards and 0 otherwise.

3.5.4 Threshold GARCH (GJR-GARCH) Model

The GJR-GARCH model was extended by Glosten, Jaganathan and Runkle, (1993). The generalized specification of GJR-GARCH (1,1)model for the conditional variance is given by:

$$h_t = \omega + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1} + \gamma \varepsilon_{t-1}^2 I_{t-1}^- \tag{22}$$

where I_{t-1}^- if $\varepsilon_{t-1} < 0$ and 0 otherwise. In GJR-GARCH model, good news is given by $\varepsilon_{t-1} > 0$, and bad news is given by $\varepsilon_{t-1} < 0$. Good news has impact of α_1, while bad news has an impact of $\alpha_1 + \gamma$. If $\gamma > 0$, bad news produces more volatility, an indication of leverage effect. If $\gamma \neq 0$, the impact of news is asymmetric. The GJR-GARCH (1,1) model with dummy variable in the conditional variance is specified as

$$h_t = \omega + \phi_1 D_1 + ... + \phi_n D_n + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 (h_{t-1}) + \gamma \epsilon_{t-1}^2 I_{t-1}^- \tag{23}$$

where $D_1, \ldots, D_n$ are the dummy variables which take the value 1 for each point of sudden change in variance onwards and 0 otherwise. Lastrapes

(1989) and Lamoreux and Lastrapes (1990) argued that when relevant random level shifts in variance are ignored in the standard GARCH variants, they tend to overestimate the persistence in volatility. Thus given the modified GARCH models which take these breakpoints identified by Bai and Perron multiple breakpoint test into consideration, the shock persistence (i.e., $\alpha_1 + \beta_1$) is predicted to be smaller than that found by the conventional GARCH models.

3.5.5 Estimation and Distributional Assumption of GARCH family Models

The estimates of GARCH process are obtained by maximizing the log likelihood function:

$$\ln(L\theta_t) = -\frac{1}{2}\sum_{t-1}^{T}(\ln 2\pi + \ln h_t + \frac{\varepsilon_t^2}{h_t}) \tag{24}$$

The five distributional assumptions employed in the estimation of parameters in this work are given by:

1. Normal (Gaussian) distribution (ND)is given by:

$$f(z) = \frac{1}{\sqrt{2\pi}}e^{\frac{-z^2}{2}}, \quad -\infty < z < \infty \tag{25}$$

and the normal distribution to the log likelihood for observation t is:

$$l_t = \frac{-\frac{1}{2}\log(2\pi) - \frac{1}{2}\log h_t - \frac{1}{2}(y_t - X_t'\theta)^2}{h_t} \tag{26}$$

2. The student- t distribution (STD) is given by

$$f(z) = \frac{\Gamma(\frac{v+1}{2})}{\sqrt{v\pi}\Gamma(\frac{v}{2})}(1 + \frac{z^2}{v})^{\frac{-(v+1)}{2}}; -\infty < z < \infty \tag{27}$$

and the student-t distribution to the log-likelihood contributions is of the form:

$$l_t = \frac{1}{2}\log\{\frac{\pi(v-2)\Gamma(\frac{v}{2})^2}{\Gamma(\frac{v+1}{2})}\} - \frac{1}{2}\log h_t - \frac{(v+1)}{2}\log\{1 + \frac{(y_t - X_t'\theta)^2}{h_t(v-2)}\} \tag{28}$$

where the degree of freedom $v > 2$ controls the tail behaviour. The t-distribution approaches the normal distribution as $v \to \infty$.

3. Skewed Student-t Distribution is given by:

$$f(z;\mu,\sigma,\nu,\lambda)=\begin{cases} bc\left(1+\frac{1}{v-2}\left(\frac{b(\frac{z-\mu}{\sigma})+a}{1-\lambda}\right)^2\right)^{-\frac{v+1}{2}}, & \text{if } z<-\frac{a}{b} \\ bc\left(1+\frac{1}{v-2}\left(\frac{b(\frac{z-\mu}{\sigma})+a}{1+\lambda}\right)^2\right)^{-\frac{v+1}{2}}, & \text{if } z\geq-\frac{a}{b} \end{cases} \tag{29}$$

where v is the shape parameter with $2<v<\infty$ and λ is the skewness parameter with $-1<\lambda<1$. The constants a,b and c are given as:

$$a=4\lambda c(\frac{v-2}{v-1}), \qquad b=1+3(\lambda)^2-a^2, \qquad c=\frac{\Gamma(\frac{v+1}{2})}{\sqrt{\pi(v-2)\Gamma(\frac{v}{2})}}$$

whereμ and σ^2 are the mean and variance of the skewed student-t distribution respectively.

4. The Generalized Error Distribution (GED) is given as:

$$f(z,\mu,\sigma,\nu)=\frac{\sigma^{-1}v\exp(-\frac{1}{2}\mid\frac{\frac{z-\mu}{\sigma}}{\lambda}\mid^{v})}{\lambda 2^{(1+(1/v))}\Gamma(1/v)} \qquad 1<z<\infty \tag{30}$$

$v>0$ is the degrees of freedom or tail -thickness parameter and $\lambda=\sqrt{2^{(-2/v)}\Gamma(1/v)/\Gamma(3/v)}$ and the GED distribution to the log-likelihood contributions is given by:

$$l_t=-\frac{1}{2}\log\{\frac{\Gamma(\frac{1}{v})^3}{\Gamma(\frac{3}{v})(\frac{v}{2})^2}\}-\frac{1}{2}\log h_t-\{\frac{\Gamma(\frac{3}{v}(y_t-X_t'\theta)^2}{h_t\Gamma(\frac{1}{v})}\}^{\frac{v}{2}} \tag{31}$$

The GED is a normal distribution if $v=2$, and fat-tailed if $v<2$.

5. (v) Skewed Generalized Error Distribution (SGED) is given as:

$$f(z;v;\xi)=v(\frac{1}{2\theta\Gamma\frac{1}{v}})exp(\frac{(z-\delta)^2}{1-sign(z-\delta)\xi\}^v\theta^v} \tag{32}$$

where $\theta = \Gamma(\frac{1}{v})^{-0.5} S(\xi) A S(\xi)^{-1}, S(\xi) = \sqrt{1 + 3\xi^2 - 4A^2\xi^2}$,
$A = \Gamma(2/v)\Gamma(1/v)^{-0.5}\Gamma(s/v)^{-0.5}$, $v > 0$ is the shape parameter controlling the height and heavy-tail of the density function while ξ is a skewness parameter of the density with $-1 < \xi < 1$.

4.0 Results and Discussion

4.1 Preliminary Data Analysis

4.1.1 Descriptive Statistics of Daily Stock Prices and Returns

To further understand the distributional properties of the stock prices and returns, summary statistics for both series are computed and results are presented in Table 1.

Table 1: Descriptive Statistics of Stock Prices and Returns

Statistic	ASI	Returns
Mean	23947.77	0.029172
Maximum	66371.00	11.26503
Minimum	4792.030	-12.54935
Standard Deviation	13316.67	1.004785
Skewness	0.656391	-0.112892
Kurtosis	3.340807	15.11793
Jarque-Bera	362.237	28920.00
P-value	0.0000	0.0000

The summary statistics shown in Table 1 indicates positive means for both daily stock prices and returns which indicate gain in the stock market for the trading period under review. The positive standard deviations for both stock prices and returns show the dispersion from the means and high level of variability of price changes in the stock market during the study period. The summary statistics also show positive asymmetry for daily stock prices (skewness = 0.656391) and negative asymmetry for the daily returns (skewness = -0.112892). The distribution is leptokurtic for both daily stock prices and daily returns as kurtosis = 3.340807 and 15.11793 respectively indicating the presence of fat-tails in the series. The distribution is non-normal for both series as Jarque-Bera statistic is 362.2371 and 28920 respectively with the marginal p-values of 0.0000 in both series.

4.1.2 Graphical Examination of Daily Stock Prices and Returns

As a first step in time series analysis, the original series (daily stock prices) as well as the transformed series (daily log returns) were plotted against time and the graphical properties of the series were observed. The plots are presented in Figures 1 and 2 respectively.

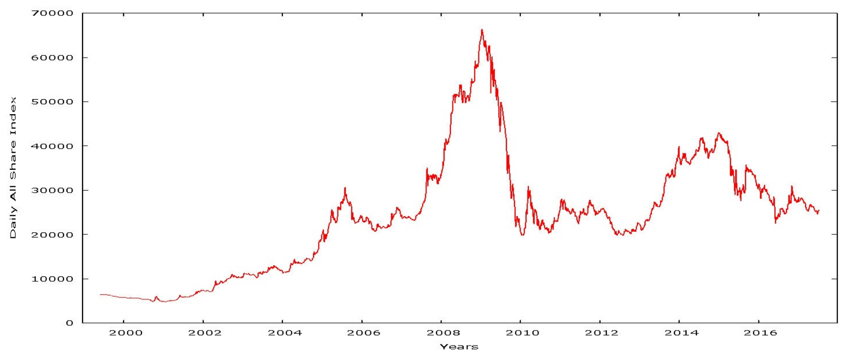

Figure 1: Time Plot of Daily Stock Prices (ASI) from 1999 – 2017.

The daily stock prices presented in Figure 1 suggests that the series has mean and variance that change with time and the presence of a trend indicating that the series is not covariance stationary. The plot of the daily return series presented in Figure 2 suggests that the series has a constant mean and variance with absence of trend indicating that it is generated by a random walk and is thus weakly stationary. The plot in Figure 2 also indicates that some periods are more clustered than others as large changes in stock returns tend to be followed by large changes and small changes are followed by small changes. This phenomenon is described as *volatility clustering*.

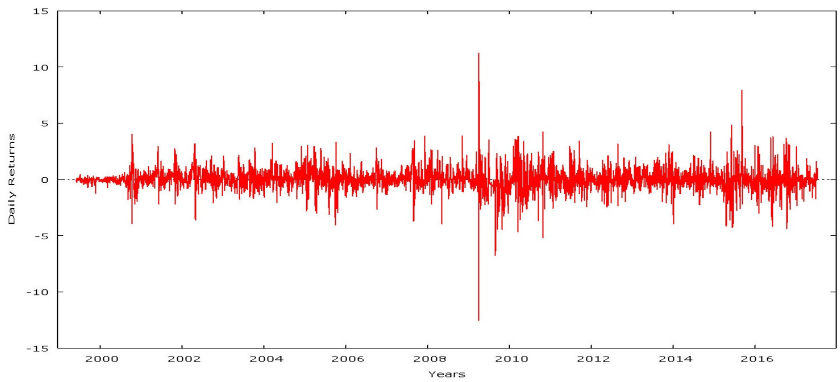

Figure 2: Time Plot of Daily Log Returns in Nigeria

Volatility clustering as one of the characteristic features of financial time series was first noticed in studies conducted independently by Mandelbrot (1963), Fama (1965) as well as Black (1976), when they observed the occurrence of large changes in stock prices being followed by large changes in stock prices of both positive and negative signs and the occurrence of small stock price changes being followed by periods of small changes in prices. Sequel to this result, numerous researchers including Poterba and Summer (1986), Tse (1991), McMillan et al. (2000), Najand (2002), Lee (2009), Emenike (2010), and Ezzat (2012) among others have in recent times documented evidence in literature proving that financial time series normally exhibit volatility clustering and leptokurtosis.

4.1.3 Unit Root and Heteroskedasticity Tests Results

Ng and Perron unit root test is employed in examining stationarity characteristics of both daily stock prices and returns in this work. The results of Ng and Perron unit root test together with heteroskedasticity test for ARCH effects are presented in Table 2.

Table 2: Ng – Perron Unit Root and Heteroskedasticity Tests Results

Variable	Option	Ng-Perron test statistics			
		MZ_α	MZ_t	MSB	MP_T
Y_t	Intercept only	-0.63183	-0.44522	0.70465	26.9371
	Intercept and trend	-3.71650	-1.26353	0.33998	23.1074
r_t	Intercept only	-2102.35*	-32.4217*	0.01542*	0.01169*
	Intercept and trend	-2213.14*	-33.2652*	0.01503*	0.04119*
Asymptotic Critical Values					
1%	Intercept only	-13.8000	-2.58000	0.17400	1.78000
5%		-8.10000	-1.98000	0.23300	3.17000
1%	Intercept and trend	-23.8000	-3.42000	0.14300	4.03000
5%		-17.3000	-2.91000	0.16800	5.48000
Heteroskedasticity Test for ARCH Effect		F-Statistic	1306.912	nR^2	1023.994
		P-value	0.0000	P-value	0.0000

Note: *denotes the significant of Ng-Perron test statistics at 1% and 5% significance levels. MZ_α, MZ_t, MSB and MP_T are the modified transformation of the standardized estimators of Z_α, Z_t, Bhargava R_1 statistic and Elliot *et al.* point optimal statistic respectively (see Phillips, 1987, Phillips and Perron, 1988, Ng and Perron, 2001, Bhargava, 198 and, Elliot *et al.* 1996).

The results of Ng – Perron unit root test reported in Table 2 indicates that the daily market prices are indeed non-stationary. This is shown by the Ng – Perron test statistics being higher than their corresponding asymptotic critical values at 1% and 5% levels. However, the Ng – Perron unit root test result of the daily stock returns show evidence of covariance stationarity as the test statistics are all smaller than their corresponding asymptotic critical values at all the designated test sizes both for constant only and for constant and linear trend. These results confirmed the result observed by visual inspection of time plots reported in Figures 1 and 2.

The lower panel of Table 2 indicates result of the residual test of heteroskedasticity for ARCH effects. The test rejects the null hypothesis of no ARCH effects in the residuals of returns. This means that the errors are time varying and can only be modeled using heteroskedastic ARCH family models.

4.2 Searching for Optimal Symmetric and Asymmetric GARCH Models

To select the best fitting symmetric and asymmetric GARCH models with suitable distributional assumption, information criteria such asAkaike information criterion (AIC) due to Akaike (1978) and Schwarz information criterion (SIC) due to Schwarz (1978)are employed in conjunction with log likelihoods (LogL). The best fitting model is one with largest log likelihood and minimum information criteria. Result is presented in Table 3.

Table 3: Model Order Selection

S/N	Model	Distribution	LogL	AIC	SIC
1	GARCH (1,1)	ND	-5745.7719	11499.5438	11525.3863
2	GARCH (1,1)	STD	-5459.1434	10928.2868	10966.5899
3	GARCH (1,1)	GED	-5462.6070	10935.2144	10967.5175
4	**GARCH (1,1)***	**SSTD**	**-5457.7421**	**10927.4842**	**10960.2479**
5	GARCH (1,1)	SGED	-5462.5949	10937.1898	10975.9535
6	EGARCH (1,1)	ND	-5700.8134	11411.6268	11443.9299
7	**EGARCH (1,1)***	**STD**	**-5432.0372**	**10876.7345**	**10915.4982**
8	EGARCH (1,1)	GED	-5437.2834	10886.5667	10925.3305
9	EGARCH (1,1)	SSTD	-5432.3433	10878.0866	10923.3110
10	EGARCH (1,1)	SGED	-5437.0913	10888.1826	10933.4070
11	GJR-GARCH (1,1)	ND	-5745.6656	11501.3311	11533.6342
12	**GJR-GARCH (1,1)***	**STD**	**-5456.5393**	**10927.0787**	**10965.8424**
13	GJR-GARCH (1,1)	GED	-5461.4886	10934.9771	10973.7409
14	GJR-GARCH (1,1)	SSTD	-5456.7168	10927.4336	10972.6580
15	GJR-GARCH (1,1)	SGED	-5461.4738	10936.9475	10982.1719

Note: * and **bold face** denotes the model selected by the search criteria. ND stands for normal distribution; STD stands for student-t distribution, SSTD stands for skewed STD; GED denotes generalized error distribution while SGED denotes skewed GED.

Table 3 shows results of 15 different symmetric and asymmetric GARCH models estimated with different innovation densities. The information criteria together with the log likelihood optimally selects symmetric GARCH (1,1) with skewed student-t distribution (SSTD), asymmetric EGARCH (1,1) and GJR – GARCH (1,1) both with student-t distribution (STD) as the best candidates to model the daily stock return volatility in Nigerian stock market.

4.3 Results of Symmetric and Asymmetric GARCH Models

The parameter estimates of the selected GARCH-type models with their respective error distributions are presented in Table 4.

Table 4: Parameter Estimates of Symmetric and Asymmetric GARCH Models

Parameter	Coefficient	Std. Error	z-statistic	P-value
Symmetric GARCH (1,1) Model with Skewed Student-t innovation				
Mean equation				
μ	-0.0004	0.0099	-0.0428	0.0059
Variance equation				
ω	0.0227	0.0058	3.8910	0.0000
α_1	0.3494	0.0331	10.5700	0.0000
β_1	0.7026	0.0248	28.3800	0.0000
v	4.8839	0.3524	13.8600	0.0000
$\alpha_1+\beta_1$	1.0520			
ARCH LM Test			0.481800	0.4876
Asymmetric EGARCH (1,1) Model with Student-t innovation				
Mean equation				
μ	-0.0105	0.0003	-33.3800	0.0000
Variance equation				
ω	-0.3670	0.0310	-15.2100	0.0000
α_1	0.4683	0.0310	15.0900	0.0000
γ	0.0331	0.0119	2.7690	0.0056
β_1	0.9444	0.0082	115.70	0.0000
v	5.0763	0.3761	13.5000	0.0000
$\alpha_1+\beta_1$	1.4127			
ARCH LM Test			0.793590	0.3731
Asymmetric GJR-GARCH (1,1) Model with Student-t innovation				
Mean equation				
μ	-0.0074	0.0084	-0.8821	0.3777
Variance equation				
ω	0.0217	0.0057	3.8420	0.0001
α_1	0.2456	0.0332	10.420	0.0000
γ	-0.0401	0.0224	-1.789	0.0436
β_1	0.8064	0.0246	28.710	0.0000
v	4.8654	0.3510	13.860	0.0000
$\alpha_1+\beta_1+\gamma/2$	1.0320	ARCH LM Test	0.556927	0.4555

The results of Table 4 shows the parameter estimates of symmetric GARCH (1,1) model with SSTD, asymmetric EGARCH (1,1) and GJR – GARCH (1,1) models both with STD. In the symmetric GARCH (1,1) model, all the parameters of the model are statistically significant. The positive and significant coefficients of the ARCH term (α_1) and GARCH term (β_1) clearly shows that stock market news about past volatility have explanatory power on current volatility. The GARCH (1,1) model showed evidence of volatility clustering in Nigerian stock market. However, the sum of ARCH term and GARCH term is greater than unity, i.e., ($\alpha_1 + \beta_1 = 1.0520 > 1$). This shows that the conditional variance is unstable, unpredictable and the entire process is non-stationary. This indicates over persistence of shocks in Nigerian stock market which can eventually explode to infinity. This result corroborates the previous findings of Bala and Asemota (2013), Adewale et al. (2016),Fasanya and Adekoya (2017) among others. Stock markets with explosive shocks are not conducive for long term investment as investors in such markets can loss or gain indefinitely.

From the results of asymmetric EGARCH (1,1) and GJR – GARCH (1,1) models, all the estimated parameters of the models are statistically significant at 5% significance levels, the parameters of the models also show over persistence of shocks as the mean reverting rates are all greater than one in both models. The existence of asymmetric response in the daily stock returns is confirmed given the non-zero asymmetric and leverage effect parameters in both models ($\gamma = 0.0331$) and ($\gamma = -0.0401$) for EGARCH (1,1) and GJR – GARCH (1,1) models, respectively. The positive and significant leverage effect parameter in the EGARCH (1,1) model indicates that positive shocks (good news) increases volatility more than negative shocks (bad news) of the same sign. The negative and significant leverage effect parameter in the GJR – GARCH (1,1) model shows that positive shocks (market advances) leads to increased volatility more than negative shocks (market retreats) of the same magnitude. Thus the study found empirical evidence for asymmetry without leverage effect.

When GARCH models are estimated using student-t and skewed student-t distributions, the t-distribution degree of freedom (shape) parameter,(v) need to be greater than 2 for the distributions to be fat-tailed. From the parameter estimates of GARCH models presented in Table 4, the shape parameter $v = 4.8839 > 2$ for GARCH model, $v = 5.07632 > 2$ for EGARCH model and $v = 4.8654 > 2$ for GJR-GARCH model indicating that the stock return series under review are fat-tailed (leptokurtic).

The heteroskedasticity (ARCH LM) tests of residuals for ARCH effects of the estimated models shown in Table 3 shows that the conditional variance equations for GARCH (1,1), EGARCH (1,1) and GJR – GARCH (1,1) models were well specified as the models captured all the ARCH effects and none is remaining in the innovations. This is clearly shown by the non-significant p-values of the F-statistics and nR^2 tests associated with the ARCH LM tests.

The estimated GARCH-type models in this work are well specified and captured all the remaining ARCH effects in the residuals, yet the shocks persistence in volatility remained very high giving rise to long memory. This is an indication that the stock return series are contaminated with structural breaks. When stock return series are contaminated with structural breaks, their volatility estimates are biased to unity, see Perron (1989, 1990). Hence there is need to investigate the presence of structural breaks in the series.

4.4 Investigating Exogenous Breaks in the Return Series

To investigate whether the return series were contaminated with structural breaks, Bai and Perron multiple breakpoints testing procedure (Bai and Perron, 1998, 2003) was applied. The structural break points in volatility with time periods were presented in Table 5. A maximum of 3 break points for daily stock returns were detected.

Table 5: Structural Breakpoints in Volatility with Time Periods

Return	Break Points	Time Periods
Daily Returns	3	10th August, 2003 – 15th December, 2004 23rd December, 2008 – 17th March, 2009 19th November, 2015 – 11th April, 2016

One of the reasons for the structural breaks was the economic recession in 2004 and the economic recovery in Nigeria in 2005, the crude oil price fluctuations in the country was another cause, the global financial crises which started in 2007 to 2009 also affected the Nigerian stock market. The terrorist attacks of Niger Delta militants, Boko Haram attacks in 2013 to date were also contributing factors; other reasons were as a result of internal, local, domestic, political or economic crises in the country.

4.5 Symmetric and asymmetric Volatility Estimates with breaks

The detected structural breaks are considered in the volatility models by incorporating indicator (dummy) variable in the conditional variance equations of the symmetric GARCH (1,1), asymmetric EGARCH (1,1) and GJR-GARCH (1,1) models. The result is reported in Table 6.

By incorporating structural break points in the volatility models, it was observed that there were significant decreases in the values of shock persistence parameters (β_1) in all estimated GARCH-type models. There were also significant reductions in the values of mean reversion rates ($\alpha_1 + \beta_1$) in all estimated models of the stock market returns. Also by including the structural breaks in these models, the stationarity and stability conditions of the models are satisfied as the sum of ARCH and GARCH terms were less than one in all the estimated models with breaks. This shows that the conditional variance process was stable and predictable and that the memories of volatility shocks were remembered in Nigerian stock market. Mean reverting and stationary stock returns were good for long term investment. This result agreed with the previous findings of Bala and Asemota (2013), Yaya and Gil-Alana (2014), Dikko *et al.* (2015), Muhammad and Shuguang (2015), Aluko et al. (2016), Adewale et al. (2016) and Kuhe and Chiawa (2017). All the estimated asymmetric models retain the asymmetric response property without the presence of leverage effect. By comparing the performance of the estimated GARCH type models, it was observed that the asymmetric EGARCH (1,1) with student-t innovation density outperformed the symmetric GARCH (1,1) and threshold GJR-GARCH (1,1) models by reducing the shock persistence in Nigerian stock market more gladly.

Table 6: Parameter Estimates of GARCH models with Structural Breaks

Parameter	Coefficient	Std. Error	z-statistic	P-value
Symmetric GARCH (1,1) Model with Skewed Student-t innovation				
Mean equation				
μ	-0.0103	0.0082	-1.2571	0.2087
Variance equation				
ω	0.0182	0.0030	5.9864	0.0000
ϕ	-0.0005	0.0074	5.5195	0.0000
α_1	0.3348	0.0253	13.2516	0.0000
β_1	0.6505	0.0140	51.3868	0.0000
v	4.8793	0.3134	15.5685	0.0000
$\alpha_1 + \beta_1$	0.9853			
ARCH LM Test			0.136101	0.7122
Asymmetric EGARCH (1,1) Model with Student-t innovation				
Mean equation				
μ	-0.0106	0.0081	-1.3031	0.0025
Variance equation				
ω	-0.3514	0.0175	-20.0806	0.0000
ϕ	--0.0007	0.0034	5.6037	0.0000
α_1	0.2479	0.0243	18.3981	0.0000
γ	0.0329	0.0126	2.6098	0.0091
β_1	0.6481	0.0064	148.245	0.0000
v	5.1639	0.3388	15.2414	0.0000
$\alpha_1 + \beta_1$	0.8960			
ARCH LM Test			0.300670	0.5835
Asymmetric GJR-GARCH (1,1) Model with Student-t innovation				
Mean equation				
μ	-0.0084	0.0083	-1.0061	0.3144
Variance equation				
ω	0.0170	0.0029	5.9175	0.0000
ϕ	-0.0016	0.0174	-18.9024	0.0000
α_1	0.2587	0.0294	11.9758	0.0000
γ	-0.0555	0.0309	-1.79352	0.0029
β_1	0.7388	0.0136	53.6464	0.0000
v	4.9171	0.3152	15.6009	0.0000
$\alpha_1 + \beta_1 + \gamma/2$	0.9698			
ARCH LM Test			0.202133	0.6530

The coefficients of the dummy variable (ϕ) was negative and statistically significant in symmetric GARCH (1,1), asymmetric EGARCH (1,1) and GJR-GARCH (1,1) models suggesting that the global financial crises, the economic recession, the crude oil price fluctuations, the Niger Delta militant and Boko Haram terrorism which impaired the stock return series had negatively affected the Nigerian stock market during the study period.

The stock return series retained the fat-tailed behaviour even after incorporating the sudden shifts in variance as the shape parameter $v = 4.8793 > 2$ for GARCH model,$v = 5.1639 > 2$ for EGARCH model and $v = 4.9171 > 2$ for GJR-GARCH model. This clearly indicated that the Nigerian stock returns were heavy-tailed, one of the stylized facts of financial returns common in developed markets.

The ARCH LM tests of residuals for ARCH effects of the estimated models with breaks shown in Table 6 indicated that the conditional variance equations for the three models have captured all the ARCH effects in the residuals of the stock return series and none was left as the p-values of the F-statistics and nR^2 tests associated with the ARCH LM tests were highly statistically insignificant.

5.0 Conclusion and Policy Implications

5.1 Conclusion

This study examined the symmetric and asymmetric responses of shocks persistence in Nigerian stock market using daily closing all share index (ASI) from Nigerian stock exchange (NSE) from 3rd July 1999 to 12th June 2017. The study modeled heteroskedasticity in Nigerian stock market by employing two different sets of specifications of symmetric GARCH (1,1), asymmetric EGARCH (1,1) and GJR-GARCH (1,1) models with varying innovation densities. The first set of models were estimated without incorporating structural breaks while the second set of estimation incorporated the detected structural breaks in the conditional variance of these models. The results of the GARCH (1,1) with skewed student-t distribution, EGARCH (1,1) and threshold GJR-GARCH (1,1) models all with student-t distributions without structural breaks showed over persistence of shocks giving rise to long memory and non-stationarity of the conditional variance process in Nigerian stock market. The estimated models were unstable with unpredictable property, however, the asymmetric models showed evidence of asymmetry without leverage effect in Nigerian stock market.

After dictating breakpoints in the return series, the second set of model estimation incorporated the structural breaks by including a dummy variable in the conditional variance equations of all the models. By incorporating the structural break points in the volatility models, there was significant decreases in the values of shock persistence parameters (β_1)and in the values of mean reversion rates ($\alpha_1+\beta_1$) in all the estimated GARCH-type models. The stationarity and stability conditions of these models were also satisfied as the sum of ARCH and GARCH terms were less than unity in all the estimated models with breaks. This also reduced the memory in the Nigerian stock market. All the estimated asymmetric models with breaks retain the asymmetric response property without leverage effect. The EGARCH (1,1) with student-t innovation density outperformed the other two competing models by reducing the shock persistence in Nigerian stock market more gladly. The choice of fat-tailed distributions by the competing models in estimating volatility confirms the presence of heavy-tails in Nigerian stock returns. This study recommends the use of asymmetric GARCH models with structural breaks in measuring volatility in Nigerian stock market as this will help to avoid over estimation of shock persistence in the conditional variance and to allow free flow of market information and wide range of aggressive trading of securities so as to increase market depth and make the Nigerian stock market less volatile.

5.2 Policy Implications

The empirical findings of this study suggest that inferences on shock persistence in volatility and long memory are more likely to be episodic and may disguise the short memory property of stock market return series with structural breaks. Consequently, caution should be exercised when inferences on shock persistence in volatility and long memory are being interpreted amid structural breaks. In addition, structural changes caused by economic crises may affect investor's financial decision in the stock market and failure to account for these structural breaks in the stock market may lead to wrong inferences and portfolio decisions by investors. Therefore, policy makers should consider the regime changes in their financial policy design.

The results of this study also characterized the Nigerian stock market with high degree of risk and uncertainty (see the work of Aluko et al., 2016). The highly explosive nature of the market further suggests that good news or bad news could have permanent effect on future periods' volatility. The higher volatile nature of Nigerian stock market may signal huge threat to both local and foreign investors; hence consistent policy reforms to install investor's confidence in the market should be implemented by government.

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